4/4/16 "Big shots are only little shots that keep shooting." -Christopher Morley

HW:

Quarter Test on Friday 4/8

AIM: What are Inverse Functions?

Warm Up:

***Exercise* #1:** Consider the two linear functions given by the formulas $f(x)=\frac{3x+7}{2}$ and $g(x)=\frac{2x-7}{3}$.

(a) Calculate $f(5)$ and $g(11)$.

$f(5)=\frac{3(5)+7}{2}=11$

$g(11)=\frac{2(11)-7}{3}=5$

(b) Calculate $f(0)$ and $g\left(\frac{7}{2}\right)$.

$f(0)=\frac{3(0)+7}{2}=\frac{7}{2}$

$g\left(\frac{7}{2}\right)=\frac{2\left(\frac{7}{2}\right)-7}{3}=0$

(c) Calculate $f(g(-1))$.

$$g(-1) = \frac{2(-1)-7}{3} = \frac{-9}{3} = -3 \qquad (-1, -3)$$

$$f(-3) = \frac{3(-3)+7}{2} = \frac{-9+7}{2} = \frac{-2}{2} = -1 \qquad (-3, -1)$$

(d) Calculate $f(g(5))$.

$$g(5) = \frac{2(5)-7}{3} = 1$$

$$f(1) = \frac{3(1)+7}{2} = 5$$

(e) Without calculation, determine the value of $f(g(\pi))$.

$$f(g(\pi)) = \pi$$

$f(x) = \frac{3x+7}{2}$

Operations:

1) Multiply by 3
2) Add 7
3) Divide by 2

$g(x) = \frac{2x-7}{3}$

Operations:

1) Multiply by 2
2) Subtract 7
3) Divide by 3

$f(x) = \frac{x}{3} + 2$ $\quad g(x) = 3x - 2$

$f(g(1)) = \quad g(1) = 3(1) - 2$

$= 1$

$f(1) = \frac{1}{3} + 2 = \frac{7}{3} \neq 1$

***Exercise* #2:** If the point $(-3, 5)$ lies on the graph of $y = f(x)$, then which of the following points must lie on the graph of its inverse?

$(5, -3)$

(1) $(3, -5)$ (3) $(5, -3)$

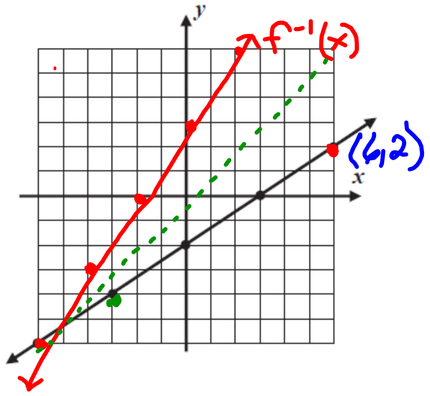
(2) $(-5, 3)$ (4) $\left(-\frac{1}{3}, \frac{1}{5}\right)$

Inverse functions have their own special notation. It is shown in the box below.

INVERSE FUNCTION NOTATION

If a function $y = f(x)$ has an inverse that is also a function we represent it as $y = f^{-1}(x)$.

***Exercise* #3:** The linear function $f(x) = \frac{2}{3}x - 2$ is shown graphed below. Use its graph to answer the following questions.



$x=6$

(a) Evaluate $\boxed{f^{-1}(2)}$ and $f^{-1}(-4)$.

when $y=2$ what is x?

$f^{-1}(-4) = -3$

(b) Determine the y-intercept of $f^{-1}(x)$.

Find x-intercept of $f(x)$ then switch values. $(0,3)$

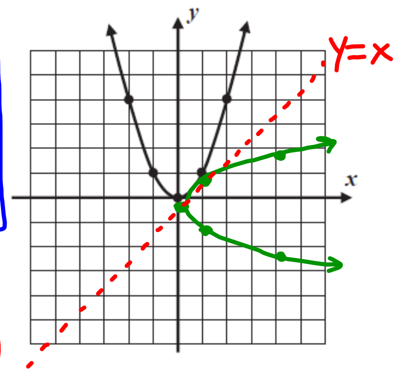
(c) On the same set of axes, draw a graph of $y = f^{-1}(x)$.

The inverse is found by reflecting the function over $y=x$

***Exercise* #4:** A table of values for the simple quadratic function $f(x) = x^2$ is given below along with its graph.

function

x	-2	-1	0	1	2
$f(x)$	4	1	0	1	4



(a) Graph the inverse by switching the ordered pairs.

x	4	1	0	1	4
$f^{-1}(x)$	-2	-1	0	1	2

(b) What do you notice about the graph of this function's inverse?

x in $f(x)$ become y in $f^{-1}(x)$

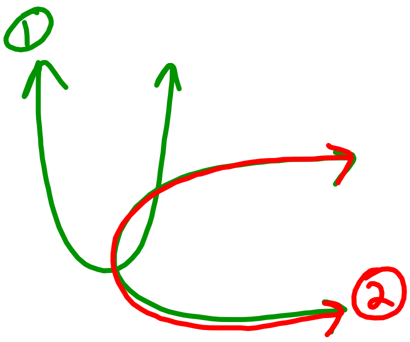
y in $f(x)$ " x in $f^{-1}(x)$

$f(x)$ reflected over $y=x$ becomes $f^{-1}(x)$.

EXISTENCE OF INVERSE FUNCTIONS

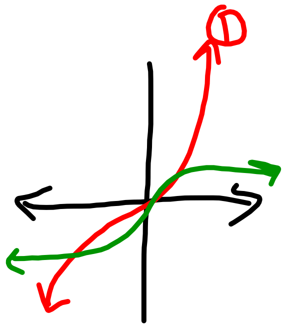
A function will have an inverse that is also a function if and only if it is one-to-one. Hence, a quick way to know if a function has an inverse that is also a function is to apply the Horizontal Line Test.

#4



① $f(x) \rightarrow$ function but NOT one-to-one

② $f^{-1}(x) \rightarrow$ not a function (Does NOT pass the vertical line test)



① $f(x) \rightarrow$ funtion & one-to-one

② $f^{-1}(x) \rightarrow$ function

Name: ______________________________ Date: ________________

INVERSE FUNCTIONS
COMMON CORE ALGEBRA II HOMEWORK

FLUENCY

1. If the point $(-7, 5)$ lies on the graph of $y = f(x)$, which of the following points must lie on the graph of its inverse?

switch x and y

(1) $(5, -7)$ (3) $(7, -5)$

(2) $\left(-\frac{1}{7}, \frac{1}{5}\right)$ (4) $\left(\frac{1}{7}, -\frac{1}{5}\right)$ ________

2. The function $y = f(x)$ has an inverse function $y = f^{-1}(x)$. If $f(a) = -b$ then which of the following must be true?

x y

(1) $f^{-1}(-b) = -a$ (3) $f^{-1}(-b) = a$

(2) $f^{-1}\left(\frac{1}{a}\right) = -\frac{1}{b}$ (4) $f^{-1}(b) = -a$ ________

x becomes y

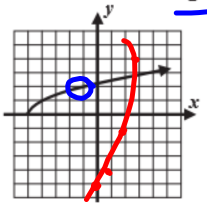
$f(x)=y$

3. The graph of the function $y = g(x)$ is shown below. The value of $g^{-1}(2)$ is

what is x when y is 2

(1) 2.5 (3) 0.4

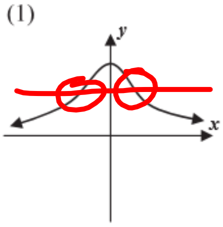
(2) -4 (4) -1 ________



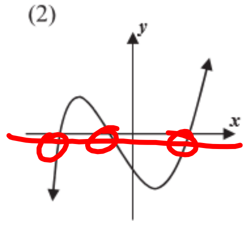
4. Which of the following functions would have an inverse that is also a function?

1to1

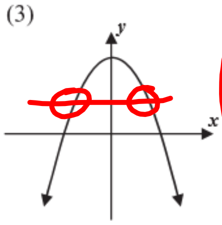
(1)



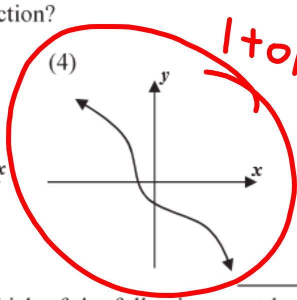
(2)



(3)



(4)



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5. For a one-to-one function it is known that $f(0) = 6$ and $f(8) = 0$. Which of the following must be true about the graph of this function's inverse?

(1) its y-intercept = 6 (3) its x-intercept = -6

(2) its y-intercept = 8 (4) its x-intercept = -8 ________

y-int

x-int
(8,0)

(0,6)

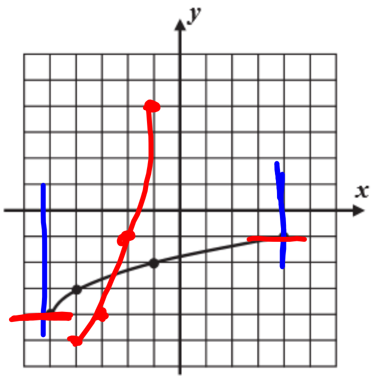
inverse (6,0) x-int

inverse (0,8) y-int

6. The function $y = h(x)$ is entirely defined by the graph shown below.

(a) Sketch a graph of $y = h^{-1}(x)$. Create a table of values if needed.

(b) Write the domain and range of $y = h(x)$ and $y = h^{-1}(x)$ using interval notation.



$y = h(x)$

Domain: $[-5, 4]$

Range: $[-4, -1]$

$y = h^{-1}(x)$

Domain: $[-4, -1]$

Range: $[-5, 4]$

APPLICATIONS

7. The function $y = A(r) = \pi r^2$ is a one-to-one function that uses a circle's radius as an input and gives the circle's area as its output. Selected values of this function are shown in the table below.

r	1	2	3	4	5	6
$A(r)$	π	4π	9π	16π	25π	36π

(a) Determine the values of $A^{-1}(9\pi)$ and $A^{-1}(36\pi)$ from using the table.

(b) Determine the values of $A^{-1}(100\pi)$ and $A^{-1}(225\pi)$.

(c) The original function $y = A(r)$ converted an input, the circle's radius, to an output, the circle's area. What are the inputs and outputs of the inverse function?

Input: Output:

REASONING

8. The domain and range of a one-to-one function, $y = f(x)$, are given below in set-builder notation. Give the domain and range of this function's inverse also in set-builder notation.

$y = f(x)$

Domain: $\{x \mid -3 \le x < 5\}$

Range: $\{y \mid y > -2\}$

$y = f^{-1}(x)$

Domain:

Range: